Cambright Solved Paper

Tags	2024 CIE IGCSE Feb/March Mathematics P4 V2
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Status	Done

1 A grocer sells potatoes, mushrooms and carrots.

(a) A customer buys 3 kg of mushrooms at \$1.04 per kg and 4 kg of carrots at \$1.28 per kg.

Calculate the total cost.

$$(3 \times 1.04) + (4 \times 1.28)$$
$$= 3.12 + 5.12$$
$$= 8.24$$

(b) In one week, the ratio of the masses of vegetables sold by the grocer is

potatoes : mushrooms : carrots = 11 : 8 : 6.

(i) Work out the mass of mushrooms sold as a percentage of the total mass.

$\frac{8}{11+8+6} \times 100$ (because we want the mushrooms out of the total)

$$= \frac{8}{25} \times 100$$
$$= 32$$

(ii) The total mass of potatoes, mushrooms and carrots sold is 1500 kg.

Find the mass of carrots the grocer sells this week.

$\frac{1500}{11+8+6}$ (total mass divided by sum of ratios) $\times 6$ (because we want only the carrots)

$= 360$

(iii) The profit the grocer makes selling 1 kg of carrots is \$0.75 .

Find the total profit the grocer makes selling carrots this week.

In (ii), we know that the grocer sold 360 kg of carrots. So the total profit is $360 \times 0.75 = 270$

(iv) On the last day of the week, the grocer reduces the price of 1 kg of potatoes by 8% to \$1.15 .

Calculate the original price of 1 kg of potatoes.

Let x be the original price

$x \times \frac{92}{100} = 1.15$ (92 because a reduction of 8% is 92%)

$x = 1.15 \times \frac{100}{92}$

$x = 1.25$

(c) The grocer buys 620 kg of onions, correct to the nearest 20 kg.
He packs them into bags each containing 5 kg of onions, correct to the nearest 1 kg.

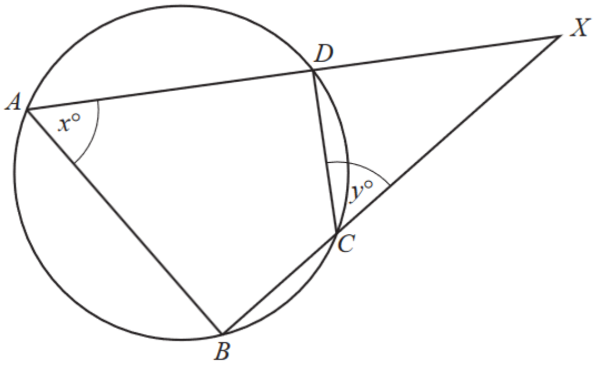
Calculate the upper bound for the number of bags of onions that he packs.

Upper bound for onions: $620 + 10 = 630$

Lower bound for mass: $5 - 0.5 = 4.5$

Upper bound for number of bags: $\frac{630}{4.5} = 140$

2



NOT TO SCALE

(a) Explain why $x = y$.
Give a geometrical reason for each statement you make.

$y + \angle \mathrm{BCD} = 180^\circ$ (angles on a straight line)

$x + \angle \mathrm{BCD} = 180^\circ$ (angles within a cyclic quadrilateral)

$\therefore x = y$

(b) Show that triangle ABX is similar to triangle CDX.

$\angle \mathrm{X}$ is the common angle for $\triangle \mathrm{ABX}$ and $\triangle \mathrm{CDX}$

$x = y$

So, the two triangles are similar (angle, angle)

(c) $AD = 15$ cm, $DX = 9$ cm and $CX = 12$ cm.

(i) Find BC.

AX(AD + DX) = 15 + 9 = 24

Since the two triangles are similar, $\dfrac{BX}{DX} = \dfrac{AX}{CX}$

$\dfrac{BX}{9} = \dfrac{24}{12}$

$BX = 18$

$BX = BC + CX$

$BC = 18 - 12 = 6$

(ii) Complete the statement.

The ratio area of triangle ABX : area of triangle CDX = ... : 1. [1]

4

The scale ratio is 2, so the area ratio is 2 squared, or 4

3 **(a)** The table shows information about the marks gained by each of 10 students in a test.

Mark	15	16	17	18	19	20
Frequency	4	1	2	1	0	2

(i) Calculate the range.

Range = highest - lowest

20 - 15 = 5

(ii) Calculate the mean.

Total $= (15 \times 4) + (16 \times 1) + (17 \times 2) + (18 \times 1) + (19 \times 0) + (20 \times 2)$

Total $= 60 + 16 + 34 + 18 + 40$

Total $= 168$

Mean $= \frac{168}{10}$ (4 + 1 + 2 + 1 + 2 = 10)

Mean $= 16.8$

(iii) Find the median.

The median will be the 5th and 6th number halved.

So $\frac{16+17}{2} = 16.5$

(iv) Write down the mode.

The mode is the number which is most frequent, so it is 15

(b) Paulo's mean mark for 7 homework tasks is 17.
After completing the 8th task, his mean mark is 17.5 .

Calculate Paulo's mark for the 8th task.

Total marks for 7 tasks = 7 × 17 = 119

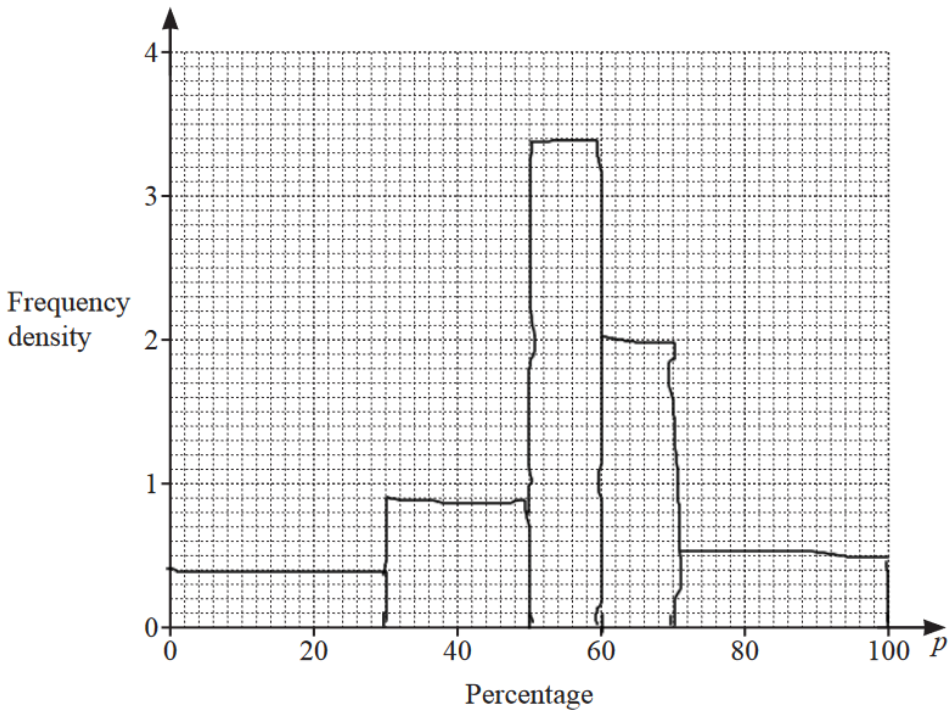
Total marks for 8 tasks = 8 × 17.5 = 140

Mark for the 8th task = 140 - 119 = 21

(c) The table shows the percentage scored by each of 100 students in their final exam.

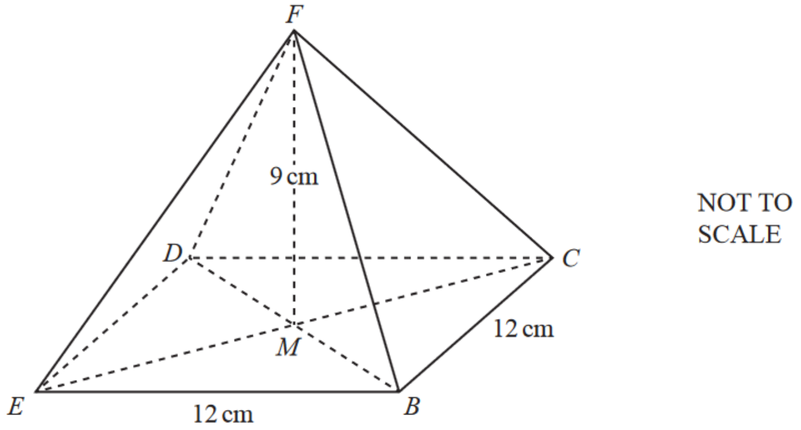
Percentage (p)	$0 < p \leqslant 30$	$30 < p \leqslant 50$	$50 < p \leqslant 60$	$60 < p \leqslant 70$	$70 < p \leqslant 100$
Frequency	12	18	35	20	15

On the grid, draw a histogram to show this information.



[4]

4 **(a)**



The diagram shows a pyramid with a square base *BCDE*.
The diagonals *CE* and *BD* intersect at *M*, and the vertex *F* is directly above *M*.
$BE = 12$ cm and $FM = 9$ cm.

(i) Calculate the volume of the pyramid.

[The volume, V, of a pyramid with base area A and height h is $V = \frac{1}{3}Ah$.]

The base area A of the pyramid is $12^2 = 144$ because it has a square base.

The height is 9

$$V = \frac{1}{3} \times 144 \times 9$$

$$V = 432$$

(ii) Calculate the total surface area of the pyramid.

The total surface area includes the base and the 4 triangles on each side.

$$A = 144 + 4(\frac{1}{2}bh)$$

We need the slant height, so using Pythagoras' theorem:

$h^2 = 6^2 + 9^2$ (half of the square width and the height make a right triangle)

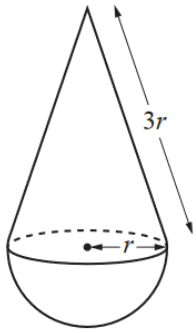
$$h = 10.82$$

$$A = 144 + 4(\frac{1}{2} \times 12 \times 10.82)$$

$$A = 144 + 259.68$$

$$A = 403.68$$

(b)



NOT TO SCALE

The diagram shows a toy made from a cone and a hemisphere.
The base radius of the cone and the radius of the hemisphere are both r cm.
The slant height of the cone is $3r$ cm.

The total surface area of the toy is $304\ \text{cm}^2$.

Calculate the value of r.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]
[The curved surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

Total surface area = 304 (cone surface + half of the sphere's surface)

$$304 = \pi r l + \frac{4\pi r^2}{2}$$

l is 3r, and the sphere's area is divided by 2 because it is only half.

$$304 = \pi r \times 3r + 2\pi r^2$$

$$304 = 3\pi r^2 + 2\pi r^2$$

$$304 = 5\pi r^2$$

$$r = \sqrt{\frac{304}{5\pi}}$$

$$r = 4.399$$

5 **(a)** **(i)** Factorise.

$$x^2 - x - 12$$

$$(x-4)(x+3)$$

(ii) Simplify.

$$\frac{x^2-16}{x^2-x-12}$$

$$\frac{x^2-4^2}{(x-4)(x+3)}$$

$$= \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x+3)}$$

$$= \frac{x+4}{x+3}$$

(b) Simplify.

$$(2x-3)^2-(x+1)^2$$

$$(4x^2-12x+9)-(x^2+2x+1)$$

$$3x^2-14x+8$$

(c) Write as a single fraction in its simplest form.

$$\frac{2x+4}{x+1}-\frac{x}{x-3}$$

$$\frac{(2x+4)(x-3)}{(x+1)(x-3)}-\frac{x(x+1)}{(x+1)(x-3)}$$

$$= \frac{2x^2-6x+4x-12}{x^2-3x+x-3}-\frac{x^2+x}{x^2-3x+x-3}$$

$$= \frac{2x^2-2x-12-x^2-x}{x^2-2x-3}$$

$$= \frac{x^2-3x-12}{x^2-2x-3}$$

(d) Expand and simplify.

$$(x-3)(x-5)(2x+1)$$

$$(x^2-5x-3x+15)(2x+1)$$

$$= (x^2-8x+15)(2x+1)$$

$$= 2x^3+x^2-16x^2-8x+30x+15$$

$$= 2x^3-15x^2+22x+15$$

(e) Solve the simultaneous equations.
You must show all your working.

$$x - 3y = 13$$
$$2x^2 - 9y = 116$$

$3y = x - 13$

$y = \frac{x-13}{3}$

Substituting this in the second equation:

$2x^2 - 9(\frac{x-13}{3}) = 116$

$2x^2 - 3x + 39 = 116$

$2x^2 - 3x - 77 = 0$

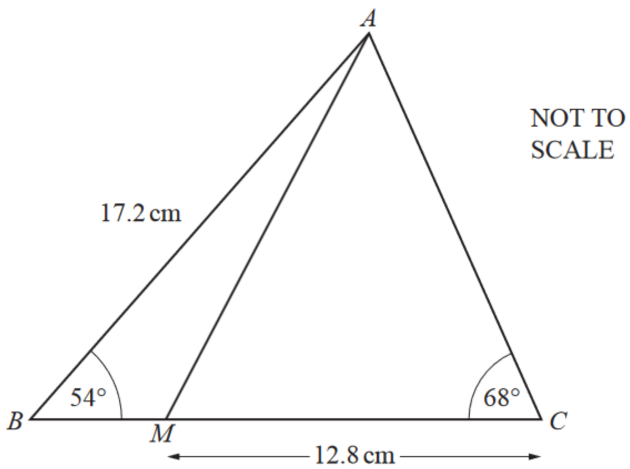
$(2x + 11)(x - 7) = 0$

$x = \frac{11}{2}$ or $x = 7$

$x = \frac{11}{3} \rightarrow y = \frac{5.5-13}{3} \rightarrow y = -2.5$ or $-\frac{5}{2}$

$x = 7 \rightarrow y = \frac{7-13}{2} \rightarrow y = -2$

6



The diagram shows triangle ABC with $AB = 17.2$ cm.
Angle $ABC = 54°$ and angle $ACB = 68°$.

(a) Calculate AC.

Using the law of sines:

$$\frac{AC}{\sin 54^\circ} = \frac{AB}{\sin 68^\circ}$$

$$AC = \frac{17.2}{\sin 68^\circ} \times \sin 54^\circ$$

$$AC = 15$$

(b) M lies on BC and $MC = 12.8$ cm.

Calculate AM.

Using the law of cosines:

$$AM^2 = MC^2 + AC^2 - 2(AC)(MC)\cos 68^\circ$$

$$AM = \sqrt{(12.8)^2 + (15)^2 - 2(12.8)(15)\cos 68^\circ}$$

Put this in your calculator in degrees mode and you should get

$$AM = 15.7$$

(c) Calculate the shortest distance from A to BC.

Shortest distance will be a straight line that forms a 90 degree angle on BC

Let A to BC = x

$\sin 54^\circ = \dfrac{x}{17.2}$ (opposite of angle over the hypotenuse)

$$x = 17.2 \times \sin 54^\circ$$

$$x = 13.9$$

7 **(a)** $\mathbf{p} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

(i) Find $3\mathbf{q}$.

$$3q = 3\begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$$3q = \begin{pmatrix} -12 \\ 15 \end{pmatrix}$$

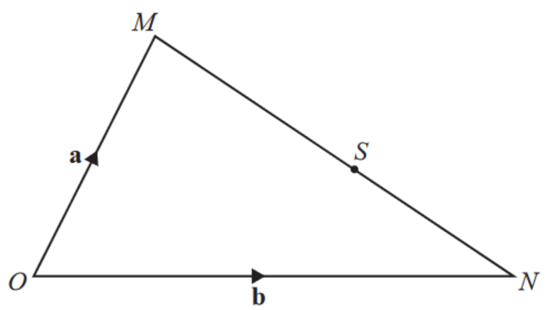
(ii) **(a)** Find $\mathbf{p} - \mathbf{q}$.

$$p - q = \begin{pmatrix} 8 \\ -5 \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 8 - (-4) \\ -5 - 5 \end{pmatrix}$$
$$= \begin{pmatrix} 12 \\ -10 \end{pmatrix}$$

(b) Find $|\mathbf{p} - \mathbf{q}|$.

$$p - q = \begin{pmatrix} 12 \\ -10 \end{pmatrix}$$
$$|p - q| = \sqrt{12^2 + (-10)^2}$$
$$|p - q| = 15.6$$

(b)



NOT TO SCALE

In triangle OMN, O is the origin, $\overrightarrow{OM} = \mathbf{a}$ and $\overrightarrow{ON} = \mathbf{b}$.
S is a point on MN such that $MS : SN = 5:3$.

Find, in terms of $\mathbf{a}$ and/or $\mathbf{b}$, the position vector of S.
Give your answer in its simplest form.

To go from O to S, we can first go from O to M, then from M to S

$$\overrightarrow{OM} = a$$

Since $MS : SN = 5 : 3$, MN has 8 parts with MS taking 5 parts and SN taking 3.

$$\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$$

$$\overrightarrow{MN} = b - a$$

Since we want MS, we want MS only,

$$\overrightarrow{MS} = \frac{5}{8}(b - a)$$

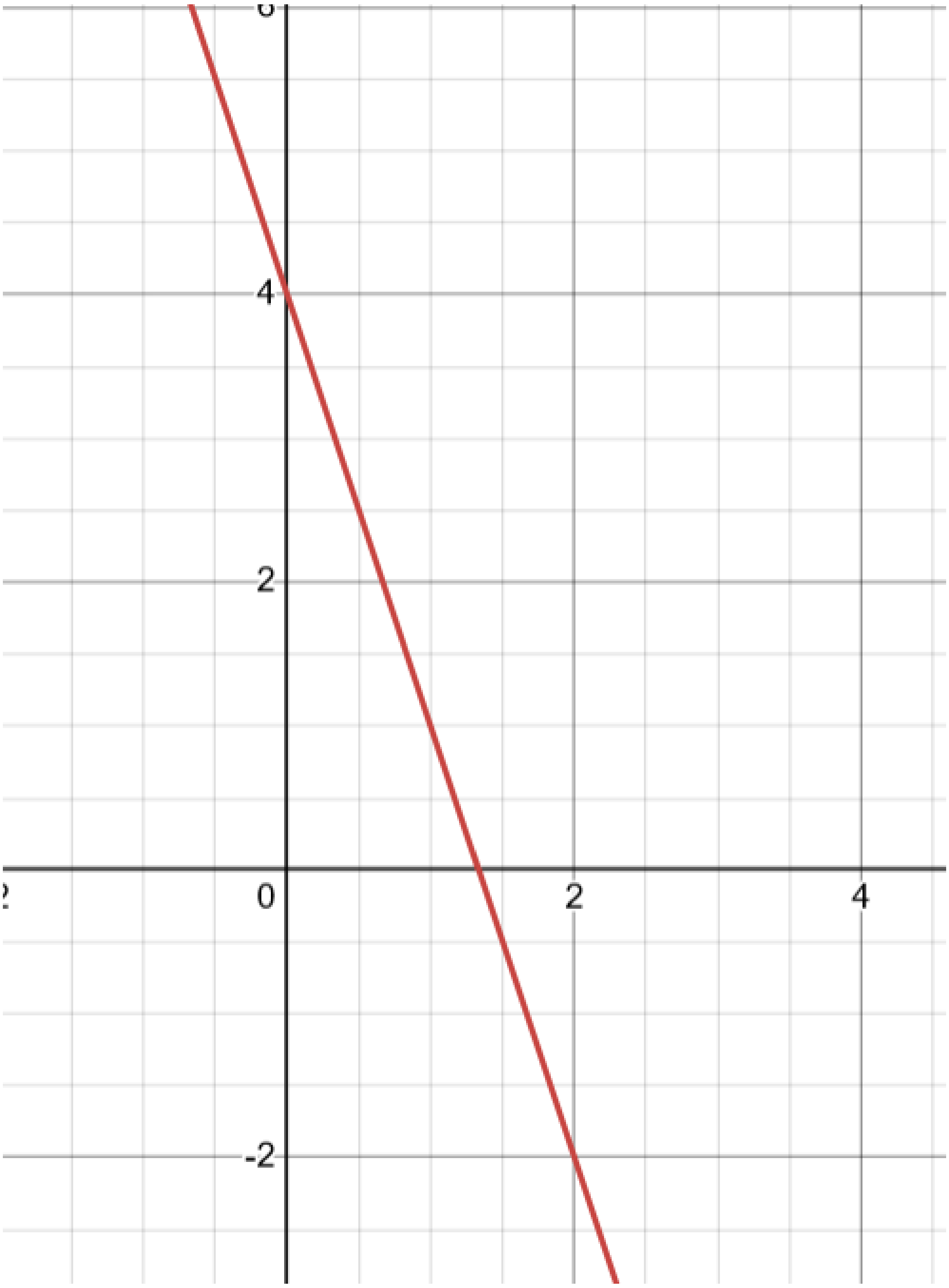
$$\overrightarrow{OS} = \overrightarrow{OM} + \overrightarrow{MS}$$

$$\overrightarrow{OS} = a + \frac{5}{8}(b - a)$$

$$\overrightarrow{OS} = a + \frac{5}{8}b - \frac{5}{8}a$$

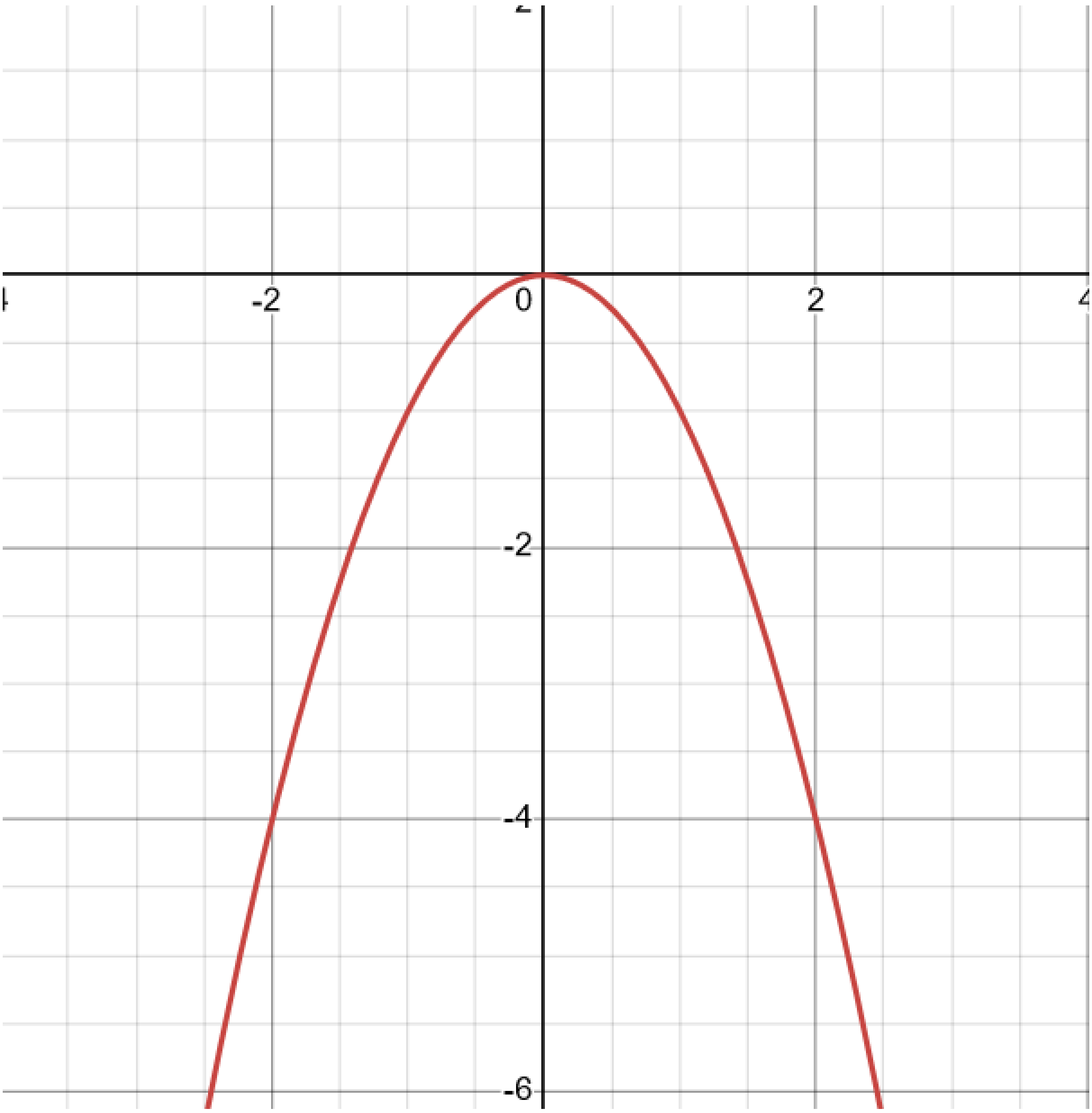
$$\overrightarrow{OS} = \frac{3}{8}a + \frac{5}{8}b$$

8 **(a)** On the axes, sketch the graph of $y = 4 - 3x$.



https://www.desmos.com/calculator/cvwtznpyk4

(b) On the axes, sketch the graph of $y = -x^2$.



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(c) **(i)** Find the coordinates of the turning points of the graph of $y = 10 + 9x^2 - 2x^3$.
You must show all your working.

If the derivative is 0, we can find the turning points

$$\frac{dy}{dx} = 18x - 6x^2$$

$$\frac{dy}{dx} = 0 \rightarrow 18x - 6x^2 = 0$$

$$6x(3 - x) = 0$$

$x = 0 \ or \ x = 3$

$x = 0 \rightarrow y = 10$

$x = 3 \rightarrow y = 10 + 9(3)^2 - 2(3)^3 = 37$

$(0,\ 10),\ (3,\ 37)$

(ii) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

Using the second derivative test:

$\frac{d^2y}{dx^2} = 18 - 12x$

$x = 0 \rightarrow \frac{d^2y}{dx^2} = 18$

$\frac{d^2y}{dx^2} > 0$

Since the second derivative is bigger than 0 here, (0, 10) is a minimum point.

$x = 3 \rightarrow \frac{d^2y}{dx^2} = 18 - 12(3)$

$\frac{d^2y}{dx^2} = -18$

$\frac{d^2y}{dx^2} < 0$

So (3, 37) is a maximum point.

9 **(a)** Janna and Kamal each invest \$8000.
At the end of 12 years, they each have \$12 800.

(i) Janna invests in an account that pays simple interest at a rate of r% per year.

Calculate the value of r.

Simple interest formula $A = P(1 + rt)$ where A is the final amount, P is the principle, r is the rate, and t is the time.

$12800 = 8000(1 + 12r\%)$

$1.6 = 1 + 12r\%$

$0.6 = 12r\%$

$r\% = 0.05$

$r = 5$

(ii) Kamal invests in an account that pays compound interest at a rate of R% per year.

Calculate the value of R.

Compound interest formula $A = P(1+r)^t$ where A is the final amount, P is the principle, r is the rate, and t is the time.

$12800 = 8000(1 + r\%)^{12}$

$1.6 = (1 + r\%)^{12}$

$\sqrt[12]{1.6} = 1 + r\%$

$r\% = 0.04$

$r = 4$

(b) The population of a city is growing exponentially at a rate of 1.8% per year.
The population now is 260 000.

Find the number of complete years from now when the population will first be more than 300 000.

Since it is growing exponentially, we must use the compound interest formula. The final amount will be 300,000 and we are finding the time t.

$260,000(1 + 1.8\%)^t > 300,000$

$(1.018)^t > 1.154$

Now doing trial and error,

$t = 8 \to (1.018)^8 = 1.153$

$t = 9 \to (1.018)^8 = 1.174$

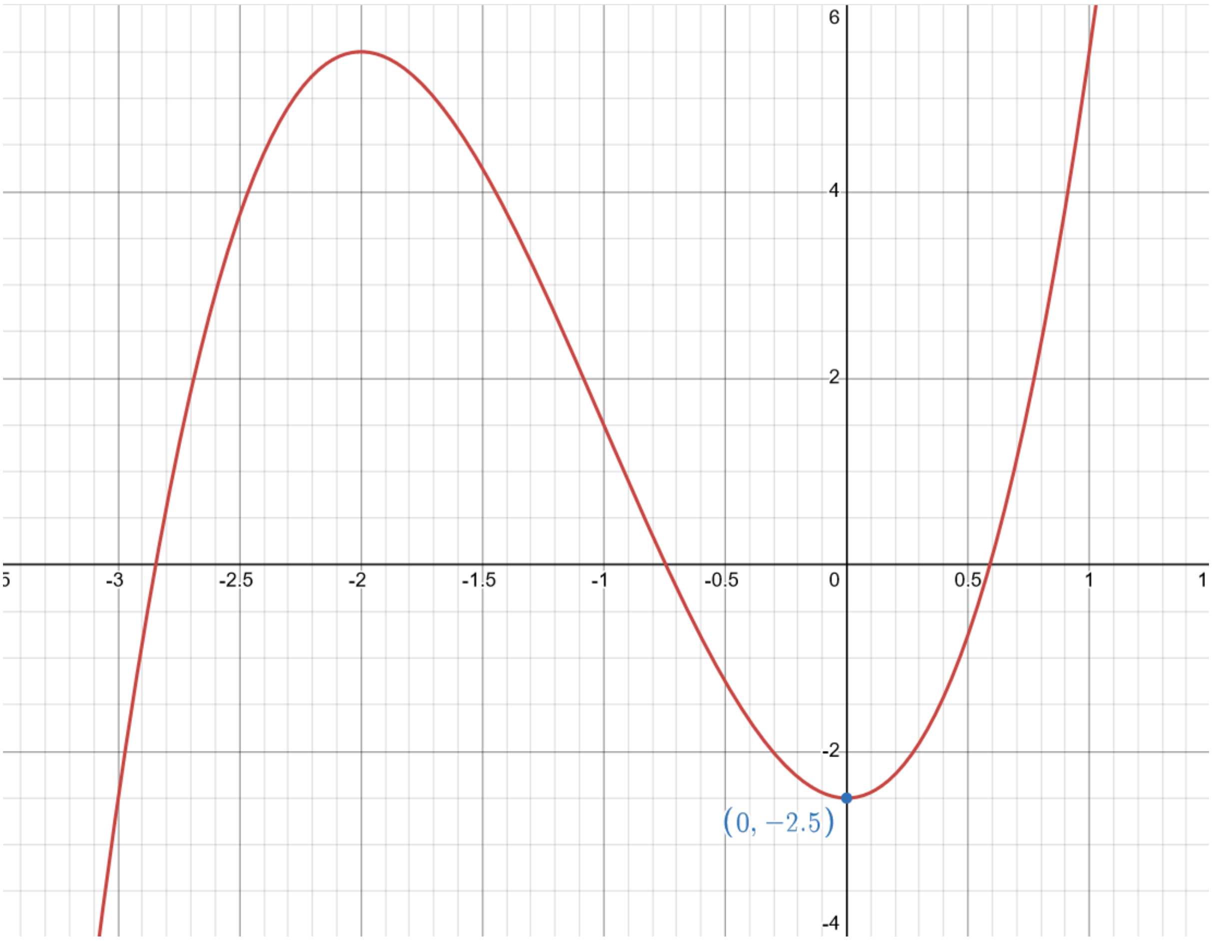
$\therefore t = 9$

10 The table shows some values for $y = 2x^3 + 6x^2 - 2.5$.

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1
y	-2.5	3.75	5.5	4.25	1.5	-1.25	-2.5	-0.75	5.5

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 2x^3 + 6x^2 - 2.5$ for $-3 \leqslant x \leqslant 1$.



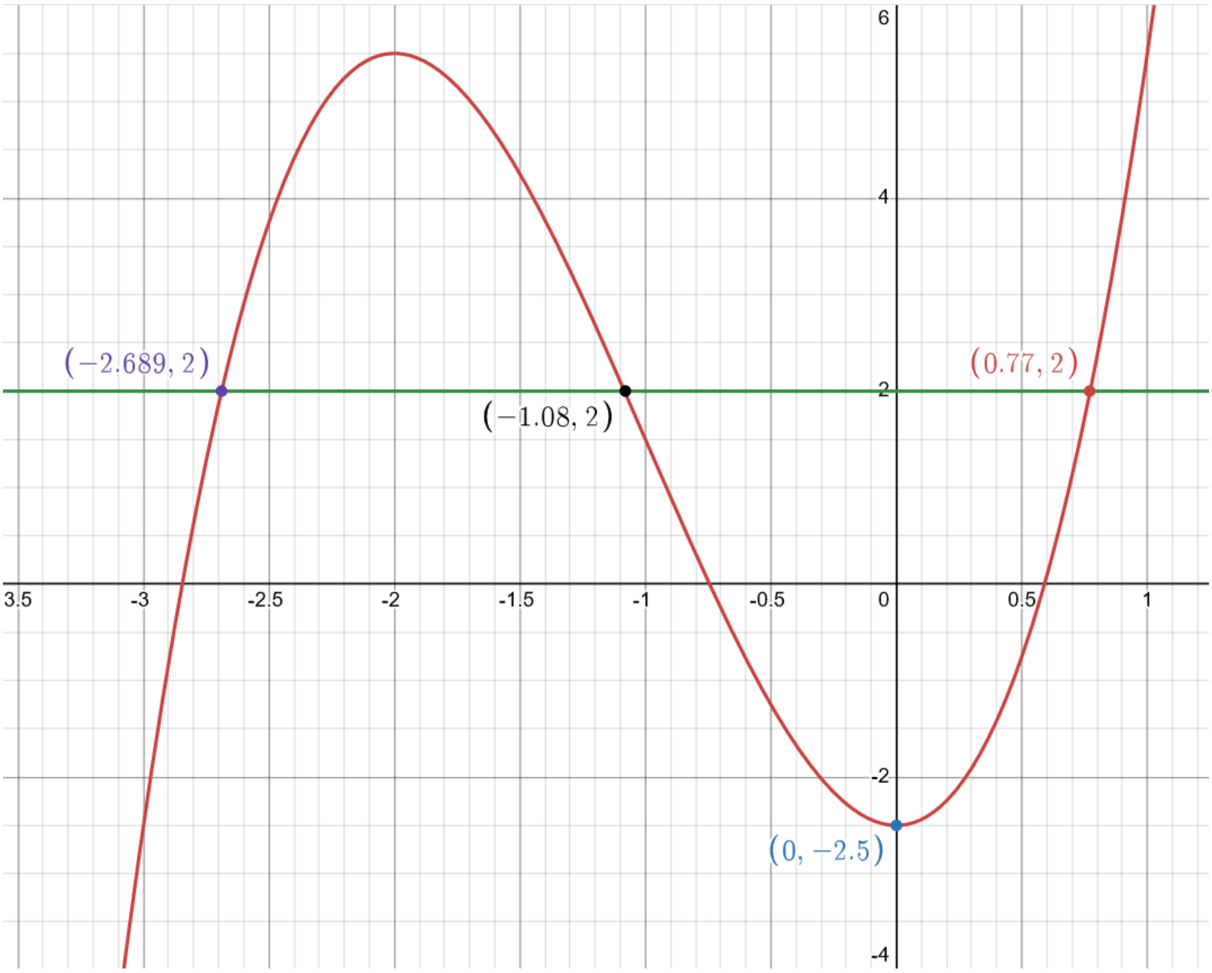
https://www.desmos.com/calculator/14rzul49si

(c) By drawing a suitable line on the graph, solve the equation $2x^3 + 6x^2 = 4.5$.

$$2x^3 + 6x^2 - 4.5 = 0$$

$$2x^3 + 6x^2 - 2.5 = 2$$

So, we draw a line at y = 2



https://www.desmos.com/calculator/14rzul49si

We can see 3 solutions at x = -2.689, x = -1.08, and x = 0.77.

(d) The equation $2x^3 + 6x^2 - 2.5 = k$ has exactly two solutions.

Write down the two possible values of k.

The place where a cubic function can have exactly 2 solutions are the turning points. So, k = -2.5 or k = 5.5

11 $\quad f(x) = \frac{1}{x}, x \neq 0 \qquad g(x) = 3x - 5 \qquad h(x) = 2^x$

(a) Find.

(i) gf(2)

$$gf(2) = g(\frac{1}{2})$$
$$= 3(\frac{1}{2}) - 5$$
$$= -\frac{7}{2}$$

(ii) $g^{-1}(x)$

$$y = 3x - 5$$
$$x = 3y - 5$$
$$3y = x + 5$$
$$y = \frac{x+5}{3}$$

(b) Find in its simplest form $g(x-2)$.

$$g(x-2) = 3(x-2) - 5$$
$$= 3x - 6 - 5$$
$$= 3x - 11$$

(c) Find the value of x when

(i) $fg(x) = 0.1$

$$fg(x) = 0.1$$
$$f(3x-5) = 0.1$$
$$\frac{1}{3x-5} = 0.1$$
$$3x - 5 = 10$$
$$3x = 15$$
$$x = 5$$

(ii) $h(x) - g(7) = 0$.

$2^x - [3(7) - 5] = 0$

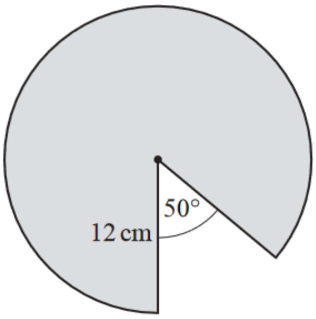
$2^x - (21 - 5) = 0$

$2^x = 16$

$x = \log_2 16$

$x = 4$

12 **(a)**



NOT TO SCALE

The diagram shows a circle of radius 12 cm, with a sector removed.

Calculate the perimeter of the remaining shaded shape.

Perimeter = circumference + 12 + 12

$= \frac{360-50^\circ}{360^\circ} \times 2\pi r + 24$

$= \frac{310^\circ}{360^\circ} \times 2\pi \times 12 + 24$

$= 88.9$

(b) The diagram in **part(a)** shows the top of a cylindrical cake with a slice removed.
The volume of cake that remains is $3510\,\text{cm}^3$.

Calculate the height of the cake.

$V = A \times h$

$3510 = A \times h$

$A = \frac{310^\circ}{360^\circ} \times \pi r^2$

$A = \frac{310^\circ}{360^\circ} \times \pi \times 12^2$

$$3510 = (\frac{310^\circ}{360^\circ} \times \pi \times 12^2) \times h$$

$$h = 9.01$$

Additional notes

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.